I. Definition of Mathematical Reasoning

Then:

> *"I am a strong advocate of math reasoning in the classroom. It's so easy for students to "get lucky," when solving a math problem. The answer could be correct, but the reasoning behind the process they used is completely false."*
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> *"Mathematical reasoning is more than just finding out where the student makes mistakes. It's about showing students that math has a purpose, that it is not about some "old person" thinking about ways to make their life horrible! Some of the basic questions that we have pursued in my class are "Why is the surface area of a cylinder: circumference×height+2×area of the base?" "When solving for linear equations, why do you have to do the same thing to both sides of the equation?" These mathematical reasoning opportunities truly give students a different way of thinking about math. In terms of the geometry example, students may still struggle with applying the formula to a problem, but at least they are able to take a manipulative, and understand what surface area actually measures in a solid object." (Note: This is not the complete Math Bio, but the paragraph that I am building upon)*

Now:

I still believe that mathematical reasoning is a way to give math a purpose and to help students understand where formulas come from. Mathematical reasoning is an argument made, to justify one's process, procedure, or conjecture, to create strong conceptual foundations and connections, in order for students to be able to process new information. Mathematical reasoning moves students beyond simply memorization of facts, towards thinking beyond the rules and procedures to forming their own questions and conjectures. Reasoning requires well-developed tasks and pre-planned questions, in order for students to communicate at the higher-levels of comprehension that mathematical reasoning requires. Mathematical reasoning is much more involved than asking "Why?"

II. Ball & Bass Article

> *"We have tried not only to illustrate what substantial mathematical reasoning by third graders can look like but also to show that these skills and practices were learned across tie, from the first day of class in September." (Bass and Ball, 43)*
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> *One key thing that resonates in my mind is that this is not an instant transformation in a classroom. Mathematical reasoning is not something that students walk into your classroom knowing how to do, and what you expect of them. Rather it is a process that must be learned while you are teaching the content. I feel strongly that this is something that needs to be started from the first day of school. Show students something that they did not know. Explore problems that most students would not automatically understand. Every day, I have a "Challenge of the Day." From the first week of school, I show students a problem that would not make sense to them at first glance. When students know that they will not know how to solve the problem automatically, they take it as what it is… a challenge. The students no longer think, "I don't know how to do this… I must be slow." Rather, students look at it as, "I want to figure this out, to show that I know what I am doing." It is so exciting*

to watch students discuss the reasoning behind what they are doing. When they think they have the answer, I always tell them the same thing, no matter if they are right or wrong… "Prove it!" I am already seeing a change in the conversations among my students. (Note: This is not the complete assignment, but the paragraph that I am building upon)

Mathematical reasoning is a process that takes many years to perfect with students. However, since I have done my research I have experienced it first hand. Students are not used to explaining their process or thinking abstractly. Rather, students <u>and</u> teachers alike must learn it. It is a skill that must be practiced all of the time, and still it will never be mastered. The transformation that Ball & Bass talks about in their article was not fully clear to me, while I read it the first time. In September, students worked on arithmetic problems and had to discuss possible solutions with each other. "This argument is empirical or inductive, not deductive. It is the kind of empirical reasoning that can increase confidence in scientific hypothesis, but it is not a mathematical proof. " (Ball & Bass, 33). Just five short months later, students transformed their communication skills to show an "idea of a general proof." (Ball & Bass, 36) Note, Ball & Bass are not making the claim that the entire class is showing clear signs of mathematical reasoning and proof making, however, the reader can see a transition that students are making.

This transformation is what I saw from my research project as well. Students were still depending on me to provide guidance and structure, but they were at least making an attempt to reason through the math. Teachers need to be aware that just because students are forming complete proofs, does not mean that their reasoning skills aren't improving. Reserving just one single chapter aside for mathematical reasoning tasks showed results were not distinct in terms of assessment scores, but students are getting more comfortable with reasoning and showed more signs of conversations that were rich is explanations.

III. What Reading Formed My Definition of Mathematical Reasoning

There were two main articles that truly solidified my definition of mathematical reasoning, for two very different reasons. First the Ball & Bass article enabled me to define mathematical reasoning and see the vitality of reasoning in the math classroom. Ball & Bass noted, "Unjustified knowledge is unreasoned and, hence, easily becomes unreasonable." (Ball & Bass, 28) Far too often, math doesn't make sense to students because it doesn't have a purpose. Mathematical reasoning gives purpose for students. But more importantly, "Mathematical reasoning can serve as an instrument of inquiry in discovering and exploring new ideas, a process that we call the *reasoning of inquiry*. Mathematical reasoning also functions centrally in justifying or proving mathematical claims, a process that we call the *reasoning of justification*, the focus of this chapter." (Ball & Bass, 30) It's impressive when you see a classroom of students who are thinking beyond the math concepts that the teacher "provides/presents," to draw their own questions and concerns. Tasks that allow opportunities for those reasoning skills to develop give students the chance to develop their inquiry based learning.

The second article that helped solidify my thoughts of mathematical reasoning and what it looks like in the classroom is "Questioning Our Patterns of Questioning" by Beth Herbel-Eisenmann and M. Lynn Breyfogle. This article pushed me to change the way that I ask students questions in my class. The one thing that I have worked on in my own classroom is to have more focused interaction in my classroom. "A focusing-interaction pattern requires the teacher to listen to students' responses and guide them based on what they students are thinking rather than how the teacher would solve the problem." (Herbel-Eisenmann and M. Lynn Breyfogle, 486) Starting with where the students have began the problem, allows them the opportunity to make sense of their own work, not the work of the teacher. By slowly evolving from focused-interactions with students, I can work on funneling their questions.

IV. Reflect Broadly on 2-3 Course Questions

3. What kinds of tasks support mathematical reasoning?
4. What classroom cultures and instructional practices help K-12 students learn to reason mathematically?

> *Note: Since it is important for students to see the connections between units, I have taken these two questions and combined them into one reflection, rather than two separate reflections.*

Transforming a classroom into one that requires heavily on mathematical reasoning tasks and conversations is a delicate and involved task. Teachers need to first create a classroom environment that is welcoming enough that students can look to each other for information, but structured enough that everyone can say what they feel, without being disrespected. The task needs to be broad enough that there are countless concepts that are explored, but specific enough that curriculum standards are being met and mastered by all students. Third, communication between all members in the classroom, including the teacher, need to begin with the students own thought process, but be constantly monitored by the teacher who is meticulously monitoring all conversations. When all these three pedagogical strategies align in one class, the results will be a math classroom that is able to productively work in a way that students are able to think beyond the required task, and "serve as an instrument of inquiry in discovering and exploring new ideas." (Ball & Bass, 30)

First, before any reasoning task can be developed, students need to feel comfortable in the classroom. Chazan lists at least five different tips/techniques that will help teachers set this environment up.

1. A classroom needs to be set up as a community-not just a collection of individuals.
2. The teacher does not need to be the wisdom holder, rather classrooms need to be built from "logic and mathematical evidence."
3. The classroom needs to focus towards mathematical reasoning instead of procedures and processes. In other words, classrooms need to focus more on higher-levels of thinking versus lower level of questions.
4. Problems need to be directed towards solving problems and less about finding the answer.
5. Problems and concepts in math need to be connected to the students, instead of a separate entity that they only hear about for 1 hour a day (in math class)

From this list, we can see that there are a large amount of areas that teachers need to reflect on, before setting up that environment. Every day, these five techniques need to be reminded to students. Putting a list on the board that says "The teacher is not the wisdom holder!" is not the trick though. Rather, this idea takes time for students to realize. For example, if a classroom is set up where the teacher answers any/all questions that a student asks, the students will feel like the teacher is the only person that has the answer. On the other hand, if the teacher asks what other students think about a question, they will slowly start looking to each other for answers. Again, this is a process that takes the entire year to "prove" to students.

Once that environment is developed, teachers need to design a task that can lead students to making connections among topics, but still cover the required curriculum. Having a task that requires students to use mathematical reasoning takes a great deal of preparation and planning. The level of the task can be categorized by two main titles; lower-level and higher-level. Lower level tasks could either be classified as memorization or procedures without connections. The results will be conversations that are highly dependant on right/wrong answers and connections are not made between math concepts. Students will not be required to think beyond the task. Higher-level tasks can be classified as "procedures with connections " or "doing mathematics." (Mary Kay Stein) These higher-levels of thinking tasks, require students to advance their math skills, to see connections among concepts.

But the success/failure of a task is not independent of all other variables. Rather the conversations that occur will show proof of reasoning taking place. These conversations, at times, depend largely on the questions that the teacher asks students. The Herbel-Eisenmann article points out three different types of questioning styles, that can be found in a class; Initiation Response-Feedback (IRF), Focusing-Interactions and Funneling. Each questioning style will have a wide-range of student response types. The purpose of some questions are to lead students down the right path, some questions are to take students from where they are at, to the next level, and other questions are to help students verbalize their thoughts.

There is not one single aspect that supersedes the other two. Rather, the combination of all three components is the only way for teachers to include successful lessons that are rich in mathematical reasoning. It is almost impossible to improve or work on one area, and ignore the other two. I found that while completing my data collection for my research question, even though I was focusing on the tasks, the environment and the questions that I asked students changed dramatically. Mathematical reasoning in a interconnected web, that if one aspect is improved, the entire system is improved.